Hall Ticket Number:

Code No. : 13119A

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) III-Semester Main Examinations, December-2018

Linear Algebra and its Applications

(Open Elective-I)

Time: 3 hours　　　　　　　　　　　　　　　　　　　　　　　　Max. Marks: 60

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Q.No.	Stem of the question	M	L	CO	PO
	Part-A (10 × 2 = 20 Marks)				
1.	In a vector space V, Prove that additive inverses are unique.	2	1	1	1
2.	Verify whether $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for the vector space $V = R^3$.	2	3	1	1
3.	If $T : P_2 \rightarrow P_2$ is a linear operator and $T(1) = 1 + x$; $T(x) = 2 + x^2$; $T(x^2) = x - 3x^2$ then find $T(-3 + x - x^2)$.	2	2	2	1
4.	Show that $[T^{-1}]_B = ([T]_B)^{-1}$ if T is an invertible linear operator on a finite dimensional vector space V and B is an ordered basis for V.	2	4	2	2
5.	Let **v** be a fixed vector in R^n and define $S = \{\mathbf{u} \mid \mathbf{u} . \mathbf{v} = 0\}$. Show that S is a subspace of R^n.	2	3	3	2
6.	Determine Whether V is an inner product space $V = R^2$; $\langle u,v \rangle = u_1v_1 - 2u_1v_2 - 2u_2v_1 + 3u_2v_2$.	2	2	3	1,2
7.	Find the orthogonal complement of W in R^n with the standard inner product W = **span** $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$.	2	2	4	2
8.	State Projection Theorem.	2	2	4	1
9.	Find the coordinates of the vector **v** relative to the ordered basis B $B = \{1, x - 1, x^2\}$ ' $v = p(x) = -2x^2 + 2x + 3$.	2	2	1	2
10.	Explain the importance of Gram-Schmidt process	2	4	3	1,2

Contd....2

Part-B (5 × 8 = 40 Marks)

11.	a)	Let a, b and c be fixed real numbers. Let V be the set of points in three-dimensional Euclidean space that lie on the plane P given by: $ax + by + cz = 0$. Define addition and scalar multiplication on V coordinate wise. Verify that V is a vector space.	4	4	1	
	b)	Let W be the subspace of M_{2x2} of matrices with trace equal to 0, and let $S=\left\{\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\right\}$. Show that S is a basis for W.	4	3	1	1,2
12.	a)	Suppose that T: V → W is a linear transformation and B = {$v_1, \ldots, v_n$} is a basis for V. If T is one-to-one, then Prove that {$T(v_1), \ldots T(v_n)$} is a basis for R(T).	4	5	2	2
	b)	Let T: $R^3 \to R^3$ be a linear operator and B = { v_1 , v_2 , v_3 } a basis for R^3. Suppose $T(v_1)=\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$ $T(v_2)=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$ $T(v_3)=\begin{bmatrix}-2\\ 2\\ 0\end{bmatrix}$ i) Determine whether $w=\begin{bmatrix}-6\\ 5\\ 0\end{bmatrix}$ is in the range T. ii) Find a basis for R (T). iii) Find dim (N(T)).	4	3	2	2
13.	a)	State and prove Cauchy-Schwartz inequality	4	2	3	2
	b)	Let V = P_2 with inner product defined by $\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)dx$. i) Show that the vectors in $S = \{1, x, \frac{1}{2}(3x^2 - 1)\}$ are mutually orthogonal. ii) Find the length of each vector in S.	4	2	3	2
14.	a)	Define an inner product on P_3 by $\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)dx$. Use the standard basis B = {v_1, v_2, v_3, v_4} = {$1, x, x^2, x^3$} to construct an orthogonal basis for P_3.	4	2	4	2
	b)	Find a basis for the orthogonal complement of W in P_2 with the inner product $\langle p,q\rangle = \int_{0}^{1} p(x)q(x)dx$, W = span {$x - 1, x^2$}	4	2	4	2

Contd... 3

	a) A subset of S of R^3 is given. $$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right\}$$ i) Find span (S) ii) Is S linearly independent?	4	2	1	2
	b) Let $f(x) = x^3$ defined on R and let $V = \{ f(x+t) \mid t \in R\}$ Define $f(x + t_1) + f(x + t_2) = f(x + t_1 + t_2)$, $cf(x + t) = f(x + ct)$ i) Determine the additive identity and additive inverses. ii) Show that V is a vector space.	4	2	1	1,2
16.	a) Let W be a subspace of an inner product space V and $B = \{ w_1, \ldots, w_m\}$ a basis for W. Then prove that vector **v** is in $W^{\perp}$ if and only if **v** is orthogonal to each vector in B.	4	4	3	2
	b) Define an inner product on P_3 by $\langle p,q \rangle = \int_0^1 p(x)q(x)dx$. Let $p(x) = x$ and $q(x) = x^2$. i) Find $\text{proj}_q p$. ii) Find $p - \text{proj}_q p$ and verify that $\text{proj}_q p$ and $p - \text{proj}_q p$ are orthogonal.	4	3	3	2
17.	Answer any ***two*** of the following:				
	a) Determine whether $B=\left\{\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -4 \end{bmatrix}\right\}$ is a basis for $M_{2\times 2}$.	4	3	1	2
	b) If $S = \{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\}$ is an orthogonal set of nonzero vectors in an inner product space V, then prove that S is linearly independent.	4	5	3	2
	c) Let V and W be finite dimensional vector spaces and $B = \{v_1, v_2, \ldots, v_n\}$ a basis for V. If $T: V \rightarrow W$ be a linear transformation, then prove that $R(T) = \text{span}\,\{T(v_1), T(v_2), \ldots, T(v_n)\}$	4	5	2	2

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

S. No.	Criteria for questions	Percentage
1	Fundamental knowledge (Level-1 & 2)	88.17
2	Knowledge on application and analysis (Level-3 & 4)	6.57
3	*Critical thinking and ability to design (Level-5 & 6) (*wherever applicable)	5.26